

Please check the examination details below before entering your candidate information

Candidate surname	Other names
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**Pearson Edexcel**  
**International**  
**Advanced Level**

Centre Number

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Candidate Number

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Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes)

Paper Reference **WMA14/01**

**Mathematics**  
**International Advanced Level**  
**Pure Mathematics P4**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2 + 5x)^3} \quad |x| < \frac{2}{5}$$

in ascending powers of  $x$ , up to and including the term in  $x^3$

Give each coefficient as a fraction in its simplest form.

(6)

①  $(2 + 5x)^{-3}$

$$2^{-3} \left(1 + \frac{5}{2}x\right)^{-3}$$

$$= 1 + (-3) \left(\frac{5}{2}x\right) + \frac{(-3)(-4)}{2!} \left(\frac{5}{2}x\right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{5}{2}x\right)^3$$

$$= 1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3$$

$$= 2^{-3} = \frac{1}{8}$$

$$\frac{1}{8} \left(1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3\right)$$

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2. A curve  $C$  has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

(5)

(b) Find an equation of the tangent to  $C$  at the point  $(3, -2)$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(2)

$$x^3 + 2xy - x - y^3 - 20 = 0.$$

$$3y - 11x + 39 = 0.$$

Implicit diff

$$3x^2 + 2y + 2x \frac{dy}{dx} - 1 - 3y^2 \frac{dy}{dx} = 0$$

$$(2x - 3y^2) \frac{dy}{dx} = 1 - 2y - 3x^2$$

$$\frac{dy}{dx} = \frac{1 - 2y - 3x^2}{2x - 3y^2}$$

(b)  $(3, -2)$

$$\frac{1 - 2(-2) - 3(3)^2}{2(3) - 3(-2)^2} = \frac{11}{3}$$

$$(3, -2) \quad m = 11/3$$

$$y = mx + c$$

$$-2 = \frac{11}{3}(3) + c \quad c = -13$$

$$y = \frac{11}{3}x - 13$$

3.

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants  $A$ ,  $B$  and  $C$

(4)

(b) (i) Hence find  $\int f(x) dx$

(ii) Find  $\int_1^2 f(x) dx$ , giving your answer in the form  $a + \ln b$ , where  $a$  and  $b$  are constants.

(6)

$$a) A(3x-1)^2 + B(x)(3x-1) + C(x) = 1.$$

let  $x=0$ .

$$A=1$$

let  $x=1/3$

$$1/3 C = 1$$

$$C=3$$

let  $x=1$

$$4A + 2B + C = 1$$

$$4(1) + 2B + 3 = 1$$

$$B = -3$$

$$(b) i) \int \frac{1}{x} - \int \frac{3}{3x-1} + \int \frac{3}{(3x-1)^2}$$

$$= \ln|x| - \frac{3 \ln|3x-1|}{3} + 3 \int (3x-1)^{-2}$$

$$= \ln|x| - \ln|3x-1| - \frac{1}{3x-1} + C$$

$$(ii) \left[ \ln|x| - \ln|3x-1| - \frac{1}{3x-1} \right]_1^2$$

$$\ln|2| - \ln|6-1| - \frac{1}{3(2)-1} = \ln\left(\frac{2}{5}\right) - \frac{1}{5}$$

$$\ln(1) - \ln(2) - \frac{1}{2} = \ln\left(\frac{1}{2}\right) - \frac{1}{2}$$

$$\left[ \ln\left(\frac{2}{5}\right) - \frac{1}{5} \right] - \left[ \ln\left(\frac{1}{2}\right) - \frac{1}{2} \right] = \ln\left(\frac{4}{5}\right) + \frac{3}{10}$$

$$a = \frac{3}{10} \\ b = \frac{4}{5}$$

4.

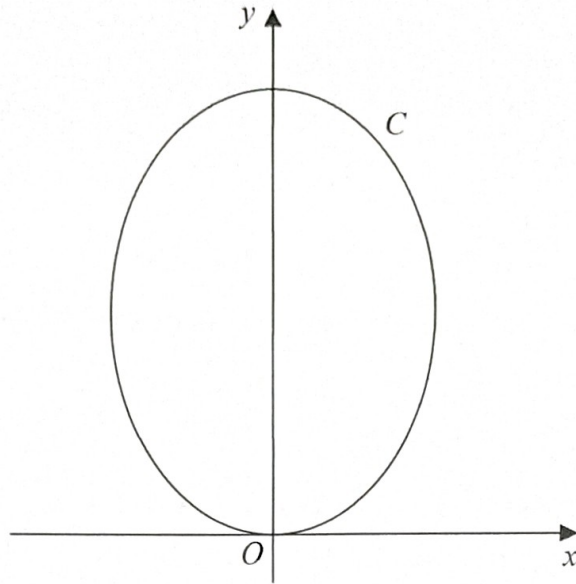


Figure 1

Figure 1 shows a sketch of the curve  $C$  with parametric equations

$$x = \sqrt{3} \sin 2t \quad y = 4 \cos^2 t \quad 0 \leq t \leq \pi$$

$$\begin{aligned} \cos 2A &= 2\cos^2 A - 1 \\ \frac{\cos 2A + 1}{2} &= \cos^2 A \end{aligned}$$

(a) Show that  $\frac{dy}{dx} = k\sqrt{3} \tan 2t$ , where  $k$  is a constant to be found.

(5)

(b) Find an equation of the tangent to  $C$  at the point where  $t = \frac{\pi}{3}$

Give your answer in the form  $y = ax + b$ , where  $a$  and  $b$  are constants.

(4)

<p>(a) <math>x = \sqrt{3} \sin 2t \quad y = 4 \cos^2 t</math></p> <p><math>\frac{dx}{dt} = 2 \times \sqrt{3} \cos 2t = 2\sqrt{3} \cos 2t</math></p> <p><math>\frac{dy}{dt}</math> of <math>\frac{4(\cos 2t + 1)}{2}</math></p> <p><math>= 2 \cos 2t + 2 \cdot</math></p> <p><math>\frac{dy}{dt} = -4 \sin 2t \cdot</math></p>	<p><math>\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-4 \sin 2t}{2 \cos 2t}</math></p> <p><math>= \frac{-2\sqrt{3} \tan 2t}{3} \quad k = \underline{\underline{-2/3}}</math></p> <p>(b) <math>\frac{-2\sqrt{3} \tan 2t}{3}</math> at <math>t = \pi/3</math>.</p> <p><math>\frac{-2\sqrt{3} \tan (2 \times \pi/3)}{3} = \underline{\underline{2}}</math></p> <p><math>x = \sqrt{3} \sin (2 \times \pi/3) = 3/2</math>.</p> <p><math>y = 4 \cos^2 (\pi/3) = 1</math>.</p>
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## Question 4 continued

$$m=2 \quad (3/2, 1)$$

$$1 = \frac{3}{2}(2) + c \quad c = -2$$

$$y = 2x - 2$$

Q4

(Total for Question 4 is 9 marks)

5.

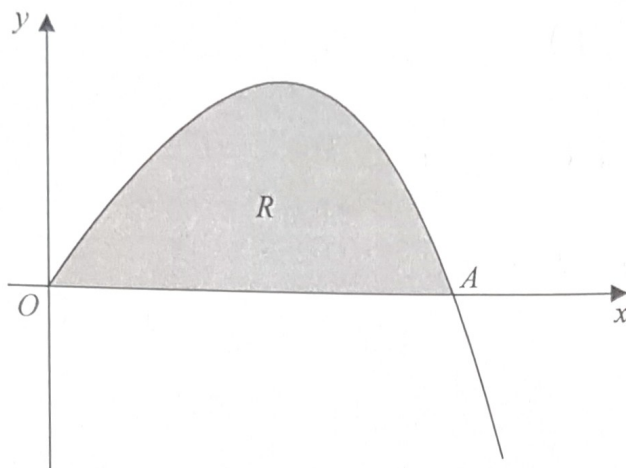


Figure 2

Figure 2 shows a sketch of part of the curve with equation  $y = 4x - xe^{\frac{1}{2}x}$ ,  $x \geq 0$

The curve meets the  $x$ -axis at the origin  $O$  and cuts the  $x$ -axis at the point  $A$ .

(a) Find, in terms of  $\ln 2$ , the  $x$  coordinate of the point  $A$ . (2)

(b) Find  $\int xe^{\frac{1}{2}x} dx$  (3)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the  $x$ -axis and the curve with equation  $y = 4x - xe^{\frac{1}{2}x}$ ,  $x \geq 0$

(c) Find, by integration, the exact value for the area of  $R$ .

Give your answer in terms of  $\ln 2$

(3)

(a)  $x$  axis ( $y=0$ )

$$0 = 4x - xe^{\frac{1}{2}x}$$

$$x(4 - e^{\frac{1}{2}x}) = 0$$

$$x = 0$$

$$4 - e^{\frac{1}{2}x} = 0$$

$$e^{\frac{1}{2}x} = 4$$

$$\ln e^{\frac{1}{2}x} = \ln 4$$

$$\frac{1}{2}x = \ln 4$$

$$x = 2 \ln 4$$

$$x = 2 \ln 2^2$$

$$x = 2 \times 2 \ln 2 = 4 \ln 2$$

## Question 5 continued

$$(b) \int x e^{1/2x} dx$$

$$u = x \quad du/dx = 1$$

$$dv/dx = e^{1/2x} \quad v = \frac{e^{1/2x}}{1/2} = 2e^{1/2x}$$

$$2xe^{1/2x} - \int 2e^{1/2x} = 2xe^{1/2x} - 4e^{1/2x} + c$$

$$(c) \int_0^{4 \ln 2} 4x - xe^{1/2x}$$

$$\frac{2 \times 4x^2}{2} - (2xe^{1/2x} - 4e^{1/2x})$$

$$\left[ 2x^2 - 2xe^{1/2x} + 4e^{1/2x} \right]_0^{4 \ln 2}$$

$$2(4 \ln 2)^2 - 2(4 \ln 2)e^{1/2(4 \ln 2)} + 4e^{1/2(4 \ln 2)} - [2(0)^2 - 2(0)e^{1/2(0)} + 4e^{1/2(0)}]$$

$$32(\ln 2)^2 - 32 \ln 2 + 16 - 4$$

$$32(\ln 2)^2 - 32(\ln 2) + 12$$



6. Prove by contradiction that, if  $a, b$  are positive real numbers, then  $a + b \geq 2\sqrt{ab}$  (4)

assumption: if  $a$  and  $b$  are positive real numbers,  $a + b \geq 2\sqrt{ab}$   
is true

$$a + b < 2\sqrt{ab}$$

$$\text{squared: } (a+b)^2 < (2\sqrt{ab})^2$$

$$a^2 + 2ab + b^2 < 4ab$$

$$a^2 - 2ab + b^2 < 0$$

$(a-b)^2 < 0 \Rightarrow$  the square of any real numbers cannot  
be smaller than 0

$\therefore$  contradiction

$\therefore$  If  $a$  &  $b$  are positive numbers,  $a + b \geq 2\sqrt{ab}$   
is true

7.

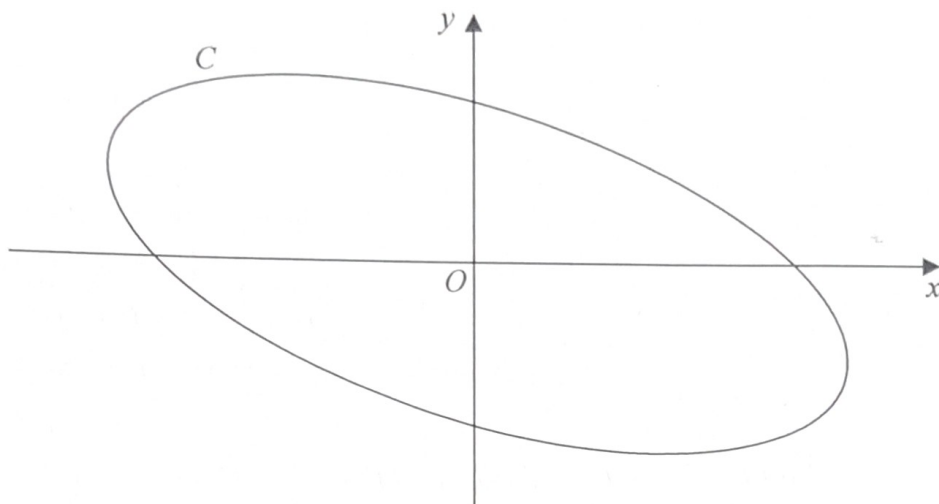


Figure 3

Figure 3 shows a sketch of the curve  $C$  with parametric equations

$$x = 4 \cos\left(t + \frac{\pi}{6}\right) \quad y = 2 \sin t \quad 0 \leq t \leq 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3} \cos t \tag{3}$$

(b) Show that a cartesian equation of  $C$  is

$$(x + y)^2 + ay^2 = b$$

where  $a$  and  $b$  are integers to be found. (2)

Handwritten solution for part (a):

$$x = 4 \cos\left(t + \frac{\pi}{6}\right)$$

$$4 \cos t \cos \frac{\pi}{6} - 4 \sin t \sin \frac{\pi}{6}$$

$$2\sqrt{3} \cos t - 2 \sin t$$

$$x = 2\sqrt{3} \cos t - 2 \sin t$$

$$x + y = 2\sqrt{3} \cos t$$

Handwritten solution for part (b):

$$\cos t = \frac{x+y}{2\sqrt{3}} \quad \sin t = \frac{y}{2}$$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$\frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$$

$$\frac{(x+y)^2}{12} + \frac{3y^2}{12} = 1$$

$$(x+y)^2 + 3y^2 = 12$$

Final result:  $(x+y)^2 + 3y^2 = 12$

8. Water is being heated in a kettle. At time  $t$  seconds, the temperature of the water is  $\theta^\circ\text{C}$ .

The rate of increase of the temperature of the water at time  $t$  is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta) \quad \theta \leq 100$$

where  $\lambda$  is a positive constant.

Given that  $\theta = 20$  when  $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t} \tag{8}$$

When the temperature of the water reaches  $100^\circ\text{C}$ , the kettle switches off.

(b) Given that  $\lambda = 0.01$ , find the time, to the nearest second, when the kettle switches off. (3)

8) (a)  $\frac{d\theta}{dt} = \lambda(120 - \theta)$

$$d\theta = \lambda(120 - \theta)dt$$

$$\frac{d\theta}{120 - \theta} = \lambda dt \quad \int \frac{1}{120 - \theta} d\theta = \int \lambda dt$$

$$\ln|120 - \theta| = \lambda t + c \quad -\lambda t = \ln|120 - \theta| + c$$

$$-\ln|120 - \theta| = \lambda t + c \quad -\lambda(0) = \ln 100 + c$$

$$c = -\ln 100 - 20$$

$$e = 100$$

$$-\lambda t = \ln|120 - \theta| - \ln 100$$

$$-\lambda t = \ln \left| \frac{120 - \theta}{100} \right| \quad e^{-\lambda t} = \frac{120 - \theta}{100}$$

$$100e^{-\lambda t} = 120 - \theta$$

$$\theta = 120 - 100e^{-\lambda t}$$

(b)  $\lambda = 0.01$

$$\theta = 100^\circ\text{C}$$

$$\ln \left( \frac{120 - 100}{100} \right) = -\lambda t$$

$$-\lambda t = -1.60943791211$$

$$\lambda t = 1.6094379121$$

$$t = \frac{1.609437912}{0.01} = 160.94$$

$$t \approx 161 \text{ s}$$

9. With respect to a fixed origin  $O$ , the line  $l_1$  is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where  $\mu$  is a scalar parameter.

The point  $A$  lies on  $l_1$  where  $\mu = 1$

- (a) Find the coordinates of  $A$ . (1)

The point  $P$  has position vector  $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line  $l_2$  passes through the point  $P$  and is parallel to the line  $l_1$

- (b) Write down a vector equation for the line  $l_2$  (2)

- (c) Find the exact value of the distance  $AP$ .

Give your answer in the form  $k\sqrt{2}$ , where  $k$  is a constant to be found. (2)

The acute angle between  $AP$  and  $l_2$  is  $\theta$

- (d) Find the value of  $\cos \theta$  (3)

A point  $E$  lies on the line  $l_2$

Given that  $AP = PE$ ,

- (e) find the area of triangle  $APE$ , (2)

- (f) find the coordinates of the two possible positions of  $E$ . (5)

a)  $\begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}$

Question 9 continued

b)  $r = a + \lambda b$

$$r = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

$$r = \begin{pmatrix} 1 + (-5)\lambda \\ 5 + 4\lambda \\ 2 + 3\lambda \end{pmatrix}$$

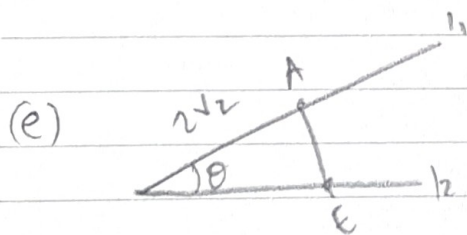
c)  $\vec{AP} = \vec{OP} - \vec{OA}$

$$\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \quad \sqrt{(-2)^2 + (2)^2}$$

$$= 2\sqrt{2} \quad \lambda = 2$$

d)  $\vec{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \quad \cos \theta = \frac{a \cdot b}{|a||b|}$

$$= \frac{16}{2\sqrt{2} \times 5\sqrt{2}} = \cos \theta = \frac{4}{5}$$



$$\cos \theta = \frac{4}{5} \quad \theta = 36.9^\circ$$

$$\frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} \times \sin 36.9^\circ = \frac{12}{5}$$

(f)  $|\vec{PE}| = 2\sqrt{2} \quad \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$

$$\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = 2\sqrt{2}$$

$$\sqrt{50\lambda^2} = 2\sqrt{2}$$

$$\lambda = \pm \frac{2}{5}$$

$$r = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 33/5 \\ 16/5 \end{pmatrix} \text{ or } \begin{pmatrix} 3 \\ 17/5 \\ 4/5 \end{pmatrix}$$